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Large extra dimensions in rare decays*

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Abstract: The prospect of experimental verification of the large extra dimension scenario in rare decays is discussed. The case of J/Ψ and Υ to photon + missing energy is calculated in detail, and it is shown that the limit on the compactification scale M_S lies at present in the ten GeV range. In contrast to the quarkonium systems, signals in Kaon and Pion decays will be small.

Keywords: Extra Large Dimensions, Rare Decays.

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Contents

1. Introduction

Recently, the possibility was discussed that the compactification-scale of large extra dimensions could lie in the TeV range [1]–[4], which would offer the possibility that their effect might be visible experimentally. For some examples using collider signatures and precision variables see [5]–[10] and references therein. The generic class of models has the quarks, leptons and other standard model fields in the usual 4 dimensions while gravity sees the other n dimensions as well. The relation between the underlying Planck scale, M_S in $n+4$ dimensions and the 4-dimensional Planck scale, M_P is then generically given in terms of the size of the extra dimension R by:

$$R^n \sim M_P^2 M_S^{-(n+2)} \,. \tag{1.1}$$

The papers mentioned above gave present and future experimental limits on the scale M_S. In this letter we will discuss what limits can be expected from rare decays using the missing energy signature.

As a generic choice we choose the compactified manifold to be an n-dimensional torus with radius R. Only the $n+4$ dimensional graviton feels this space. The general formalism needed is discussed in [11]. The lowest order coupling of the 4-dimensional states from the $n+4$-dimensional graviton, the 4-dimensional graviton, $\tilde{h}_{\mu\mu'}$, the dilaton, $\tilde{\phi}$, and their Kaluza-Klein excitations (KK), is model-independent and can be written in terms of the energy-momentum tensor. This approximation is non-renormalizable and the resulting expressions can only be used as an effective theory. Some loop-diagram calculations have been performed as well, e.g. [10, 12].

Rare decay measurements take place at much lower momenta than the collider limits discussed earlier. The main question is whether the larger precision obtainable in these experiments can compensate for the lower scales involved. Let us first look at a generic decay of a particle with mass M_D and check what we could expect on dimensional grounds. Phase space factors are of similar magnitude so we neglect these in this argument. The couplings of gravitons and KK-excitations are all proportional to $\kappa = \sqrt{16\pi G_N}$, a decay-width is thus generically suppressed by a factor of $\kappa^2 M_D^2$ compared to the usual decays. For scales $R \gg 1/M_D$ there is an additional enhancement factor due to the total number of KK-excitations that exists. For the simple compactification discussed above this number is [11]:

$$N_{m_{\tilde{h}} \leq M_D} = \int_0^{M_D^2} dm^2 \frac{R^n m^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)} \,. \tag{1.2}$$

Therefore a generic branching fraction is of order:

$$R^n \kappa^2 M_D^{n+2} \approx \left(\frac{M_D}{M_S}\right)^{n+2} , \tag{1.3}$$

Notice that this argument neglects all other dimensionful factors except phase-space going into the other decays, these can of course only be included for specific decays and we discuss the case of quarkonium in detail below. Eq. (1.3) indicates that the sensitivity to large extra dimensions is stronger when heavy mesons are probed, thus motivating to study heavy quarkonia like J/Ψ and Υ. In addition their main decay is 3-body and proportional to α_S^3 while the signal is two-body and proportional to α, providing an extra enhancement factor for the quarkonia decays. Eq. (1.3) shows also that the sensitivity decreases the more extra dimensions are 'active'.

The impact of large extra dimensions discussed here follows the formalism of [11] throughout. The possible existence of universal torsion-induced interaction from large extra dimensions [13] might lead to four-quark vertices which could enhance the sensitivity of rare meson decays to large extra dimensions significantly.

In the next section we present some arguments mainly based on angular momentum and helicity conservation why we expect, in contrast to heavy quarkonia, Kaon and Pion decays to be less promising candidates. Some of these arguments are also applicable to B and D-decays. In section 3 we discuss quarkonium decays into photon + KK-excitation in detail. We present explicit results for J/Ψ and Υ, including the total branching ratios and the photon energy spectrum.

2. Kaon and pion decays

2.1 Angular momentum and helicity

A generic problem with decays to gravitons is that they are spin two. As a consequence for most decays there needs to be a component of angular momentum in the final state as well. This leads to typical additional suppression coming from the matrix elements. This argument is not quite so strong for the dilaton which has spin-0, but since a lot of the options require the other particle to be a photon similar arguments apply.

This is the case for example in π^0 decays where calculations of $\pi^0 \to \gamma\tilde{\phi}$ and $\pi^0 \to \gamma\tilde{h}$ from triangle diagrams in quark models all give zero to leading order.

2.2 K to π KK-excitation

Weak K decays are in first approximation well described by chiral lagrangians expressed in terms of point-like meson fields with the generic structure:[1]

$$\sum_{ij} \left(a_{ij} \partial_\mu M_i \partial^\mu M_j - m_{ij}^2 M_i M_j + \mathcal{O}(M_i^3) \right) . \tag{2.1}$$

Here M_i is a generic particle field. The KK-excitation couplings are then obtained by calculating the energy momentum tensor from eq. (2.1).

As we can always diagonalize the quadratic terms in eq. (2.1), no $K \to \pi$ transition exists and thus the leading order contribution to $K \to \pi\tilde{\phi}$ or $K \to \pi\tilde{h}$ vanishes as well. Decays with more particles in the final state are of course permitted. A possible nonvanishing source comes from Penguin-like diagrams with external gravitons attached. We can expect the extra loop factors involved in this case to provide extra suppressions.

2.3 A naive argument for B and D decays

The above argument of minimally coupled KK-excitations is also present when we consider them as coming from underlying $B \to P$, with P some particle, transitions. On the other hand, since momenta in the final state here are much larger this might not lead to quite so strong suppressions as in the Kaon-Pion case.

Contributions from gravitonic penguins are however subject to the same suppressions as normal Penguins so we can expect them to be at most comparable to the $b \to s\gamma$ transition as a starting point.

3. Quarkonium

We now calculate the decays of quarkonium to a photon and a KK-excitation. We restrict ourselves to the lowest S-wave states and treat the quarks in the static approximation. An overview of this type of calculations can be found in [14]. The precise formalism of helicity projections that we use is that of [15]. In this approximation the quarks are considered to be at rest and thus no extra angular momentum

[1] Higher order terms can be brought in a similar form near the mass-shell by using the equations of motion

can be produced. The angular momentum and helicity arguments of the previous section thus apply as well and we find consequently that the spin zero states, η_c and η_b do not decay to photon-KK-excitation to leading order.

We have chosen decays including a photon since they provide a clean signature and are unambiguously calculable. Hadronic decays require at least two-gluons in addition to the graviton so they will be of relative order α_S^2/α with extra color and phase-space factors. We do not expect them to be of a very different order of magnitude than the ones we calculated.

3.1 The calculation

The diagrams contributing to the quarkonium decay into a photon and a KK-excitation are given in figure 1. For definiteness, we concentrate here in the calculation on J/Ψ decay. The replacements necessary for the calculation of the Υ decay are obvious.

The kinematics is the annihilation reaction of the quarkonium state, $\mathrm{Q\bar{Q}}$ into a photon, γ and a graviton $\tilde{h}$ or dilaton $\tilde{\phi}$. Basically, this is the reaction $\mathrm{Q}(q)+\bar{\mathrm{Q}}(\bar{q}) \rightarrow \gamma(k_\gamma) + (\tilde{h}, \tilde{\phi})(k_{\mathrm{gr}})$ for quarks at rest projected on the quarkonium state. Using the projection on charmonium states, as described in [15], and the Feynman rules for the

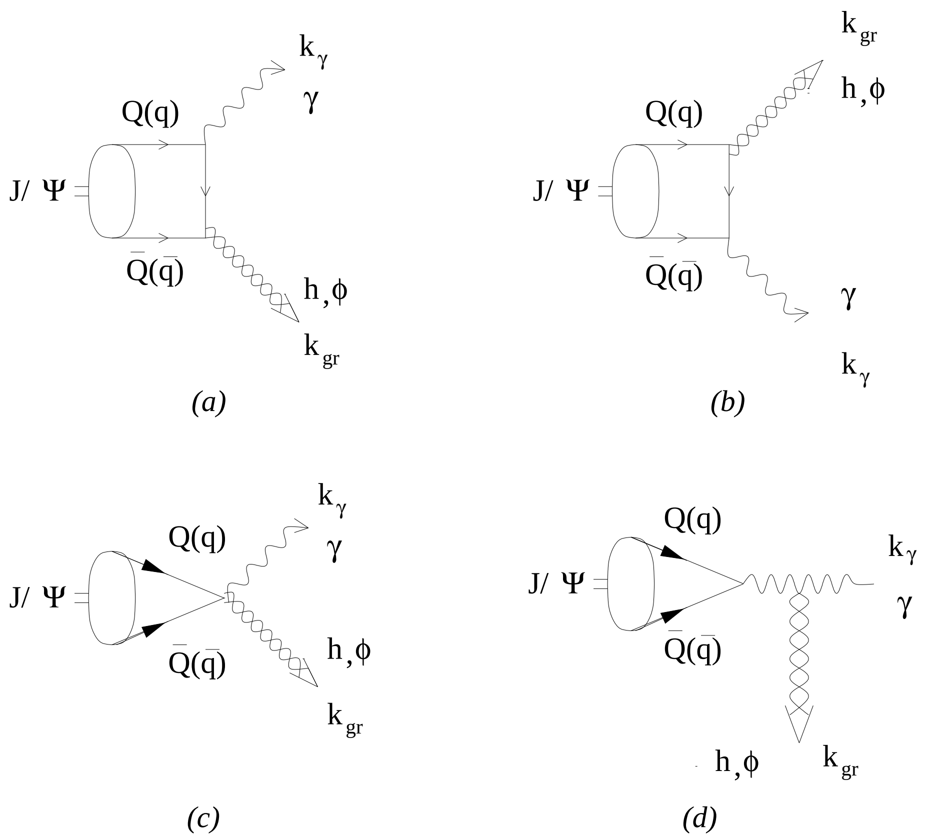

Figure 1: Diagrams contributing to quarkonium decays into a photon and a KK-excitation.

coupling of the KK-gravitons with massive fermions [11], we get for the spin-2 KK contribution:

$$
\begin{aligned}
\Psi^{L_z S_z}_{(a)\mu\mu'\nu}(\mathbf{q}) &= -i\frac{N_c e e_q \kappa}{2\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q})\bar{q}_{\mu'}\frac{\mathrm{Tr}\left[(\not{q}+m_c)\not{\varepsilon}(S_z)(\bar{\not{q}}-m_c)\gamma_\mu(\not{k}_{\mathrm{gr}}-\bar{\not{q}}+m_c)\gamma_\nu\right]}{(k_{\mathrm{gr}}-\bar{q})^2-m_c^2} \\
\Psi^{L_z S_z}_{(b)\mu\mu'\nu}(\mathbf{q}) &= i\frac{N_c e e_q \kappa}{2\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q})q_{\mu'}\frac{\mathrm{Tr}\left[(\not{q}+m_c)\not{\varepsilon}(S_z)(\bar{\not{q}}-m_c)\gamma_\nu(\not{q}-\not{k}_{\mathrm{gr}}+m_c)\gamma_\mu\right]}{(k_{\mathrm{gr}}-q)^2-m_c^2} \\
\Psi^{L_z S_z}_{(c)\mu\mu'\nu}(\mathbf{q}) &= -i\frac{N_c e e_q \kappa}{2\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q})\,\mathrm{Tr}\left[(\not{q}+m_c)\not{\varepsilon}(S_z)(\bar{\not{q}}-m_c)\gamma^\rho\right]\eta_{\mu\nu}\eta_{\mu'\rho} \\
\Psi^{L_z S_z}_{(d)\mu\mu'\nu}(\mathbf{q}) &= 2i\frac{N_c e e_q \kappa}{2\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q})\frac{\mathrm{Tr}\left[(\not{q}+m_c)\not{\varepsilon}(S_z)(\bar{\not{q}}-m_c)\gamma^\rho\right]}{(2m_c)^2}\times \\
&\quad\times\left[Q\cdot k_\gamma\eta_{\mu\rho}\eta_{\mu'\nu}-\eta_{\mu\nu}Q_{\mu'}(k_\gamma)_\rho-\eta_{\mu\rho}Q_\nu(k_\gamma)_{\mu'}+\eta_{\rho\nu}Q_\mu(k_\gamma)_{\mu'}\right], \qquad (3.1)
\end{aligned}
$$

For definiteness we have written the quark mass as m_c and used $2m_c = M_\Psi$, with Ψ the quarkonium state. The superscript indicates the orbital and spin z-component and the subscripts are the indices coupling to the KK-graviton, $\mu\mu'$ and to the photon, ν. $Q = q + \bar{q}$ and $\Psi_{LL_z}(\mathbf{q})$ is the quarkonium wave function with orbital angular momentum L and its z-component L_z. The quark and antiquark momentum are on-shell with spatial component $\mathbf{q}$ and $-\mathbf{q}$ respectively. $\varepsilon(S_z)$ is the polarization amplitude for the quarkonium state. For η_c and η_b, $\not{\varepsilon}(S_z)$ should be replaced by γ_5. For the sum of all four amplitudes we write:

$$
\Psi^{L_z S_z}_{\mu\mu'\nu}(\mathbf{q}) = \Psi^{L_z S_z}_{(a)\mu\mu'\nu}(\mathbf{q}) + \Psi^{L_z S_z}_{(b)\mu\mu'\nu}(\mathbf{q}) + \Psi^{L_z S_z}_{(c)\mu\mu'\nu}(\mathbf{q}) + \Psi^{L_z S_z}_{(d)\mu\mu'\nu}(\mathbf{q})\,, \qquad (3.2)
$$

In the static limit $\mathbf{q} \to 0$ the amplitudes simplify further and we get, summing over the photon and graviton polarizations, for the squared amplitude of the J/Ψ ($^1S_3$ state):

$$
\begin{aligned}
|M^{(J/\Psi)}_{\tilde{h}}|^2 &= \int\frac{d^3\mathbf{q}}{(2\pi)^3}\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\Psi^{Lz=0S_z}_{\mu\mu'\nu}(\mathbf{q})\left(\Psi^{L_z=0S_z}_{\mu_1\mu_1'\nu'}(\mathbf{q}')\right)^* B^{\mu\mu',\mu_1\mu_1'}\left(-\eta^{\nu\nu'}\right) \\
&= \frac{N_c}{4}\alpha_{\mathrm{em}}\kappa^2 R_0^2 e_q^2 m_c \begin{cases} 4\dfrac{m_{\mathbf{n}}^2}{m_c^2}\,; & \text{if } S_z = 0 \\ \dfrac{8}{3}+\dfrac{m_{\mathbf{n}}^4}{m_c^4}\,; & \text{if } S_z = \pm 1\,. \end{cases} \qquad (3.3)
\end{aligned}
$$

We have chosen the spin quantization axis parallel to the photon momentum. The polarization tensor of the spin-2 KK states is given by [11]:

$$
B^{\tilde{h}}_{\mu\nu,\rho\sigma} = \tilde{\eta}_{\mu\rho}\tilde{\eta}_{\nu\sigma} + \tilde{\eta}_{\mu\sigma}\tilde{\eta}_{\nu\rho} - \frac{2}{3}\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\rho\sigma}\,, \qquad \tilde{\eta}_{\mu\nu} = \left(\eta_{\mu\nu} - \frac{(k_{\mathrm{gr}})_\mu(k_{\mathrm{gr}})_\nu}{m_{\mathbf{n}}^2}\right). \qquad (3.4)
$$

The normalization of the J/Ψ wave function is given by:

$$
\int\frac{d^3\mathbf{q}}{(2\pi)^3}\Psi_{00}(\mathbf{q}) = \frac{R_0}{\sqrt{4\pi m_c N_c}}\,, \qquad (3.5)
$$

The constant R_0 describes the S-wave function at the origin. For the spin-0 KK state (dilaton) the amplitudes read:

$$\Psi^{L_zS_z}_{(a)\nu\, ij}(\mathbf{q}) = -i\delta_{ij}\frac{N_c e e_q \kappa\omega}{\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q}) \times$$
$$\times \frac{\mathrm{Tr}\left[(\not{q}+m_c)\not{\epsilon}(S_z)(\not{\bar{q}}-m_c)\left(-\frac{3}{2}\not{\bar{q}}+\frac{3}{4}\not{k}_{\mathrm{gr}}-2m_c\right)(\not{k}_{\mathrm{gr}}-\not{\bar{q}}+m_c)\gamma_\nu\right]}{(k_{\mathrm{gr}}-\bar{q})^2-m_c^2}$$
$$\Psi^{L_zS_z}_{(b)\nu\, ij}(\mathbf{q}) = -i\delta_{ij}\frac{N_c e e_q \kappa\omega}{\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q}) \times$$
$$\times \frac{\mathrm{Tr}\left[(\not{q}+m_c)\not{\epsilon}(S_z)(\not{\bar{q}}-m_c)\gamma_\nu(\not{q}-\not{k}_{\mathrm{gr}}+m_c)\left(\frac{3}{2}\not{q}-\frac{3}{4}\not{k}_{\mathrm{gr}}-2m_c\right)\right]}{(k_{\mathrm{gr}}-q)^2-m_c^2}$$
$$\Psi^{L_zS_z}_{(c)\nu\, ij}(\mathbf{q}) = i\delta_{ij}\frac{3}{2}\frac{N_c e e_q \kappa\omega}{\sqrt{2}(2m_c)}\Psi_{LL_z}(\mathbf{q})\,\mathrm{Tr}\left[(\not{q}+m_c)\not{\epsilon}(S_z)(\not{\bar{q}}-m_c)\gamma_\nu\right]$$
$$\Psi^{L_zS_z}_{(d)\nu\, ij}(\mathbf{q}) = 0\,, \tag{3.6}$$

For the sum of all three nonzero amplitudes we write:

$$\Psi^{L_zS_z}_{\nu\, ij}(\mathbf{q}) = \Psi^{L_zS_z}_{(a)\nu\, ij}(\mathbf{q}) + \Psi^{L_zS_z}_{(b)\nu\, ij}(\mathbf{q}) + \Psi^{L_zS_z}_{(c)\nu\, ij}(\mathbf{q})\,, \tag{3.7}$$

Then the amplitude squared for the spin-0 KK state (dilaton) for the the J/Ψ ($^1S_3$ state) is given by:

$$|M_{\tilde{\phi}}|^2 = \int\frac{d^3\mathbf{q}}{(2\pi)^3}\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\Psi^{L_z=0S_z}_{\nu\, ij}(\mathbf{q})B^{\tilde{\phi}}_{ij,i'j'}\left(\Psi^{L_z=0S_z}_{\nu'\, i'j'}(\mathbf{q}')\right)^*\left(-\eta^{\nu\nu'}\right)$$
$$= 2(n-1)N_c\alpha_{\mathrm{em}}\kappa^2\omega^2R_0^2e_q^2m_c\,, \tag{3.8}$$

for $S_z = \pm 1$ and zero for $S_z = 0$. The expression $B^{\tilde{\phi}}_{ij,i'j'}$ comes from the dilaton propagator and has the form:

$$B^{\tilde{\phi}}_{ij,i'j'} = \frac{1}{2}\left(P^{\vec{n}}_{ii'}P^{\vec{n}}_{jj'} + P^{\vec{n}}_{ij'}P^{\vec{n}}_{ji'}\right)\,, \tag{3.9}$$

And the projectors $P^{\vec{n}}_{ii'}$ are defined through:

$$P^{\vec{n}}_{ii'} = \delta_{ii'} - \frac{n_in_{i'}}{\vec{n}^2}\,, \qquad P^{\vec{n}}_{ij}P^{\vec{n}}_{jk} = P^{\vec{n}}_{ik}\,, \qquad P^{\vec{n}}_{ii} = n-1\,, \tag{3.10}$$

A simple check is that all amplitudes given above vanish for the η_c. The decay rate for the (unpolarized) J/Ψ is then given averaging over the initial possible polarization and the summation over the KK modes of masses $m_\mathbf{n}$ as described in [11] and eq. (1.2):

$$\Gamma(J/\Psi\to\gamma+gr)_{\tilde{h},\tilde{\Phi}} = \frac{1}{3}\sum_{S_z}\frac{1}{2M_{J/\Psi}}\sum_{\mathbf{n}}\int\frac{d^4k_\gamma}{(2\pi)^3}\int\frac{d^4k_{gr}}{(2\pi)^3}\delta^+(k_\gamma^2)\delta^+(k_{gr}^2-m_{gr}^2)\times$$
$$\times(2\pi)^4\delta(k_{J/\Psi}-(k_\gamma+k_{gr}))|M(m_\mathbf{n}^2,S_z)_{\tilde{h},\tilde{\Phi}}|^2$$
$$= \frac{1}{3}\sum_{S_z}\frac{1}{32\pi m_c^2}\int_0^{(2m_c)^2}\frac{dm_\mathbf{n}^2R^nm_\mathbf{n}^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)}\left(\frac{(2m_c)^2-m_\mathbf{n}^2}{4m_c}\right)\times$$
$$\times|M(m_\mathbf{n}^2,S_z)_{\tilde{h},\tilde{\Phi}}|^2\,, \tag{3.11}$$

The + sign over the delta functions means that the energy must be positive. The decay width becomes finally:

$$\Gamma(J/\Psi \to \gamma + gr)_{\tilde{\phi}} = (n-1)\frac{\alpha_{em}\kappa^2\omega^2 R_0^2 N_c e_q^2}{24\pi m_c}\int_0^{(2m_c)^2}\frac{dm_{\mathbf{n}}^2 R^n m_{\mathbf{n}}^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)}\left(\frac{(2m_c)^2 - m_{\mathbf{n}}^2}{4m_c}\right)$$

$$\Gamma(J/\Psi \to \gamma + gr)_{\tilde{h}} = \frac{\alpha_{em}\kappa^2 R_0^2 N_c e_q^2}{192\pi m_c}\int_0^{(2m_c)^2}\frac{dm_{\mathbf{n}}^2 R^n m_{\mathbf{n}}^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)}\left(\frac{(2m_c)^2 - m_{\mathbf{n}}^2}{4m_c}\right)\times$$
$$\times\left[\frac{8}{3} + 2\left(\frac{m_{\mathbf{n}}}{m_c}\right)^2 + \left(\frac{m_{\mathbf{n}}}{m_c}\right)^4\right], \tag{3.12}$$

The photon spectrum can be easily deduced from the formulas given above using the kinematical relation $m_{\mathbf{n}}^2 = M_{J/\Psi}^2 - 2M_{J/\Psi}E_\gamma = (2m_c)^2 - 4m_c E_\gamma$. The integrals are all polynomials and can be done explicitly.

3.2 Numerical results

For the numerical results we set $\alpha_{\rm em} = 1/137$, $e_q = 2/3$, $\omega = \sqrt{2/3(n+2)}$, $2m_c = M_{J/\Psi} = 3097$ MeV, $\kappa = \sqrt{16\pi G_N}$, and the gravitation constant $G_N = 6.70711 \times 10^{-45}$ MeV^{-2}. The value for R_0 can be obtained in the accuracy of our calculation from the tree level width for the decay $J/\Psi \to e^+e^-$ [16]:

$$\Gamma(J/\Psi \to e^+e^-) = \frac{4\alpha_{\rm em}^2 e_q^2 N_c}{3M_{J/\Psi}^2}R_0^2 = 5.2374\,\text{keV}\,, \tag{3.13}$$

This relation receives sizable QCD corrections which is the reason for the fact that the value of R_0 is not precisely known, see e.g. the discussion in [17]. Similar corrections can be expected for the decays into photons and KK-excitations. These corrections will affect the precise values of the numbers discussed below, but not any of the conclusions.

We can now define a critical compactification scale, when the contribution from graviton radiation becomes as big as the precision reached in today's experiments looking at rare J/Ψ decays. From the data given in [16] we expect that a Branching ratio of:

$$B(J/\Psi \to \gamma + \tilde{h}, \tilde{\phi}, R_{\rm crit}) = 10^{-5} \tag{3.14}$$

would be easily measurable. We used $\Gamma_{J/\Psi} = 87\,\text{keV}$ to normalize the branching ratios. The resulting $R_{\rm crit}$, converted into a limit on M_S using eq. (1.1) with $M_P = \sqrt{1/G_N}$, is shown in figure 2 as a function of n. The limits from $J/\Psi \to \gamma\tilde{h}$ and $J/\Psi \to \gamma\tilde{\phi}$ are shown as well as the similar limits from $\Upsilon \to \gamma\tilde{h}$ and $\Upsilon \to \gamma\tilde{\phi}$. For the latter ones we used $M_\Upsilon = 9.460\,\text{GeV}$, $\Gamma(\Upsilon \to e^+e^-) = 1.323\,\text{keV}$, $\Gamma_\Upsilon = 52.5\,\text{keV}$ [16], $e_q = -1/3$ and a measurable branching ratio of 10^{-4}. It is seen that in general the limit on M_S is larger for the Υ decay than for the J/Ψ decay and that the spin-2 KK state dominates over the spin-0 KK state (dilaton). In general the critical values for M_S lie in the ten GeV range.

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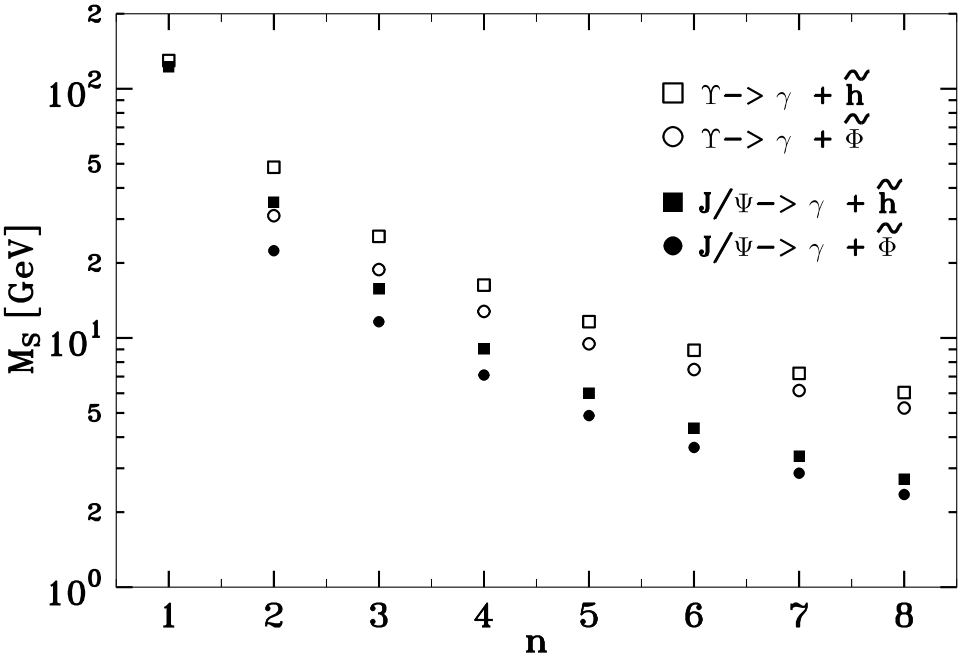

Figure 2: The lower limits on the scale M_S from the decays $J/\Psi \to \gamma\tilde{h}$, $J/\Psi \to \gamma\tilde{\phi}$, $\Upsilon \to \gamma\tilde{h}$ and $\Upsilon \to \gamma\tilde{\phi}$ as a function of the number of extra dimensions n.

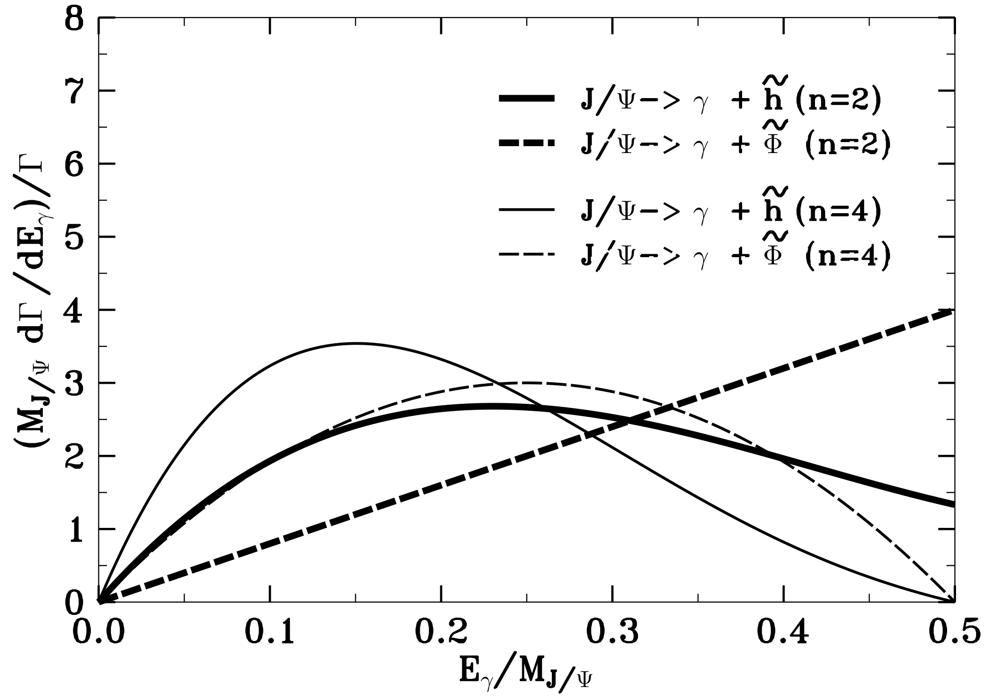

Figure 3: The photon spectrum for the dilaton $\tilde{\phi}$ and graviton $\tilde{h}$ case. Shown are $n = 2, 4$.

The photon spectrum as a function of $E_\gamma/M_{J/\Psi}$ is shown in figure 3 for the spin-2, $\tilde{h}$, and spin-0 case $\tilde{\phi}$. It shows a characteristic shape for $\tilde{h}$ and $\tilde{\phi}$ depending on n. Notice that the relative spectrum shown is neither dependent on R nor on $M_{J/\Psi}$.

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Experiment	$M_S(n=2)$	$M_S(n=4)$	$M_S(n=6)$
DELPHI	1250.0	790.0	590.0
$\Upsilon \to \gamma + \tilde{h}$	48.4	16.3	8.9

Table 1: Limits on M_S in GeV from missing-energy processes. The numbers for DELPHI are taken from [18].

4. Summary and conclusions

We have presented general arguments why looking for large extra dimensions in rare meson decays in cases where the missing energy is carried away by KK-excitations of the graviton in $4+n$-dimensions is most promising for heavy quarkonia systems. These included general dimensional arguments, angular momentum conservation and arguments due to the pointlike structure at relevant scales.

As an example, we computed J/Ψ and Υ decays into a photon and KK-excitations. The limits to be expected from these processes lie in the GeV range. The spectrum of the photon-energy E_γ in those decays is characteristic for the number n of extra large dimensions and different for spin-2 and spin-0 KK states. In general, the spin-2 KK graviton yields larger contributions than the spin-0 dilaton. Υ decays are more favorable than J/Ψ-decays due to the larger mass involved and the possibility of improving the branching limit considerably at the B-factories.

One can compare the limits on M_S in rare decays with the present best limits from e^+e^- colliders given by the DELPHI collaboration, see table 1. (Limits from other present experiments, see [18] for details, are of the same order.) The values from DELPHI are better by one or two orders of magnitude than what can be obtained from rare decays with present limits. However, our rare decays test mainly the couplings to heavy quarks while the high-energy colliders test mainly couplings to gauge bosons and light fermions, so qualitatively there is a difference between what is tested in rare decays and what is tested by lepton or even hadron colliders like the LHC. Moreover one should take into account that through possible improvements at B-Factories the limits on large extra dimensions could be possibly increased by an order of magnitude.

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